

# Prove it in Swahili: Language as an Axiomatic System

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How can students get experience in formal proof when their mathematical abilities or experiences are limited? One technique that I've been able to use successfully involves viewing human languages (or non-human languages like Klingon or Sindarin) as axiomatic systems.

Axiomatic systems may be described as consisting of four major components: definitions, axioms, theorems and an underlying logic that provides rules for deriving theorems from axioms and other theorems. When we use language systems as examples of axiomatic systems, we have definitions and an underlying logic that includes system-specific rules of inference, i.e. the rules for moving from definitions to theorems. If we think of languages as axiomatic systems we can view sentences as theorems in the system resulting from the correct application of the rules of inference.

Students are often more comfortable with words than with symbols. This is particularly true of students who have weak mathematics backgrounds or who are (or intend to be) fine arts or humanities majors in college. Using restricted subsets of human languages, students can develop skills in formal proof independently of their facility with mathematical ideas. In the following paragraphs I will illustrate the use of language to teach conjecture and proof. First, I'll show how to introduce formal proof using Swahili and then I'll describe how students can form their own conjectures and then prove them using Quiche.

Swahili is a language spoken in East Africa by more than 45 million people. I

use a small subset of Swahili (Perrott, 1978) to introduce the axiomatic method to my students.

We begin with a definition. The definitions we make are simple categorizations of strings of letters. These Swahili strings obviously have meaning, but we don't need to know what the strings of letters mean in order to prove theorems.

Definition SD1. A *nom* is any of the strings **su**, **tabu**, **atu**, **kapu**, and **jiko**.

(If we are comfortable with set notation then we could say instead that a *nom* is any element of the set {**su**, **tabu**, **atu**, **kapu**, **jiko**}.)

Students need to understand that **su** is a *nom* because it is defined to be a *nom* and that any assertion about **su** must ultimately refer back to this definition. For many students this surface arbitrariness of definition is an important component of proof to which little attention is paid.

Next we will specify a rule of inference. Essentially, a rule of inference allows us to write new strings of symbols from given strings. For our language subsets we use constructive rules of inference. They will tell us what we are permitted to do with strings. Here is a rule of inference for Swahili.

SR1. One may attach the string **ki** or the string **vi** to the left of any *nom*. The result is a theorem.

This rule of inference is simple and unambiguous. As a result, it is easily used in proving theorems. Theorems result from definitions, axioms and other theorems by means of the rules of inference. In our language fragments, the theorems will be new strings of symbols. We can now state and prove the following theorem. The justification for each step in the proof is given parenthetically.

Swahili Theorem 1 (ST1). **kisu**.

Proof.

1. **su** is a *nom* (Definition SD1)
2. **kisu** (Rule of inference SR1 from #1)

We can also prove this theorem.

ST2. **visu**.

Proof.

1. **su** is a *nom* (Definition SD1)
2. **visu** (Rule of inference SR1 from #1)

Students have no trouble stating and proving the Swahili theorems **vitabu** and **kijiko** and many others. They may also be able to calculate how many theorems can result from these rules!

These theorems and proofs are strictly formal. The student does not have to deal with the meaning of the Swahili words at all. Using a language fragment devoid of semantics forces the student to focus on the structure of the argument.

Now let's add another definition and modify our rule of inference to handle a new set of symbols.

Definition SD3. An *ajem* is any of the strings **kubwa**, **dogo**, **refu**, **zuri**, and **baya**.

SR1 (version 2). One may attach the string **ki** or the string **vi** to the left of any *nom* or to the left of any *ajem*. The result is a theorem.

We can now prove the theorem.

ST5. **kikubwa**

Proof.

1. **kubwa** is an *ajem* (Definition SD2)
2. **kikubwa** (Rule of inference SR1 from #1)

Theorem proving is an important mathematical skill, but so is theorem-conjecture. With the Swahili axiomatic system we have so far, we can ask students to formulate and prove new theorems. Almost every student will easily state and prove a theorem like **vidogu**.

They will also be able to tell that **vivi** and **kubwaki** are not theorems and explain why they are not. Specifically, **vivi** is not a theorem because no part of it is either a *nom* or an *ajem*. **kubwaki** is not a theorem because the **ki** is attached to the right of **kubwa** instead of the left as required by SR1. These simple exercises reinforce the idea that a theorem must be deduced from definitions or axioms using rules of inference.

Our rule of inference may also be expressed as an if-then statement like this.

SR1 (version 3). If  $x$  is a *nom* or  $x$  is an *ajem* then **kix** and **vix** are theorems.

Now let's add some complexity to our system with the addition of a new rule of inference. The symbol  $\triangle$  indicates an obligatory space.

SR2. If  $x$  is a *nom* and  $y$  is an *ajem* then both **vix $\triangle$ viy** and **kix $\triangle$ kiy** are theorems.

We can now state and prove the theorems.

ST6. **vitabu vikubwa**.

Proof.

1. **tabu** is a *nom*. (SD1)
2. **kubwa** is an *ajem*. (SD2)
3. **vitabu vikubwa** (SR2 from #1 and #2)

ST7. **kikapu kizuri.**

Proof.

1. **kapu** is a *nom.* (SD1)
2. **zuri** is an *ajem.* (SD2)
3. **kikapu kizuri.** (SR2 from #1 and #2).

Students will be able to conjecture and prove many other theorems using these rules and definitions.

Although, we have been stressing the formal aspect of language, we mustn't disregard the obvious fact that these Swahili theorems mean something. Students will be keen to know what the Swahili words mean in English. The theorems we have been proving are simple Swahili noun phrases. Number is indicated by the prefixes **ki-** (singular) and **vi-** (plural). In Swahili the adjective follows the noun it modifies.

Here are the adjectives with English translation: **kubwa** (*big*), **dogo** (*small*), **refu** (*long*), **zuri** (*good*), **baya** (*bad*). Here are the nouns: **su** (*knife*), **tabu** (*book*), **atu** (*shoe*), **kapu** (*basket*), **jiko** (*spoon*).

Thus, **vitabu vikubwa** may be translated into English as “the big books” and **kikapu kizuri** may be translated as “the good basket”.

We can make good use of the English translations of foreign language phrases to give students practice in another important aspect of mathematical theorem proving – conjecture. I'll illustrate how this can be done with a language from Central America, Quiche.

Quiche is a language spoken by about 650,000 people in Guatemala (Mondloch, 1978). It is one of the languages of the ancient Maya. Students can develop the analytical skills needed to formulate theories by analyzing words, phrases, and sentences in an unfamiliar language. To help students acquire these skills, I present a series of questions that will lead them to formulating their own axiomatic system.

Here are several Quiche sentences with their English equivalents.

- |                        |                           |
|------------------------|---------------------------|
| 1. <b>k'ek lē tz'i</b> | <i>The dog is black.</i>  |
| 2. <b>sak lē tz'i</b>  | <i>The dog is white.</i>  |
| 3. <b>cak lē tz'i</b>  | <i>The dog is red.</i>    |
| 4. <b>k'ek lē che'</b> | <i>The tree is black.</i> |

If I ask my students what the Quiche word for “dog” is, they have no trouble deducing from these data that the answer is **tz'i**. They are equally adept at discovering that the Quiche word **k'ek** means “black.” Furthermore, given the additional data

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|------------------------|----------------------------|
| 5. <b>rax lē che'</b>  | <i>The tree is green.</i>  |
| 6. <b>k'an lē che'</b> | <i>The tree is yellow,</i> |

my students usually have enough information to form informal conjectures about the Quiche sentences.

In particular, they see that the adjective comes first. The adjective is followed by this mysterious **lē**, which is followed, in turn, by the noun. They may not be able to articulate their conjecture explicitly at this point, but they can create new Quiche sentences on demand. For example, if asked to write “The tree is white” in Quiche, most will successfully produce **sak lē che'**.

Next, I present a larger vocabulary of Quiche words. The table below is an example.

Quiche	English	Quiche	English
<b>ja</b>	<i>house</i>	<b>balam</b>	<i>jaguar</i>
<b>tz'iquin</b>	<i>bird</i>	<b>caxlan</b>	<i>foreign</i>
<b>nim</b>	<i>big</i>	<b>cumatz</b>	<i>snake</i>
<b>co</b>	<i>strong</i>	<b>chak'</b>	<i>cooked</i>
<b>ch'uti'n</b>	<i>small</i>	<b>joron</b>	<i>cold</i>
<b>tinamit</b>	<i>town</i>	<b>lej</b>	<i>tortilla</i>
<b>jujub</b>	<i>mountain</i>	<b>lol</b>	<i>grasshopper</i>
<b>chicop</b>	<i>animal</i>	<b>patzapic</b>	<i>shaggy</i>
<b>utiw</b>	<i>coyote</i>	<b>abaj</b>	<i>rock</i>
<b>quej</b>	<i>horse</i>	<b>bak</b>	<i>thin</i>

Working in groups, the students must decide whether or not each of the following is a valid Quiche sentence. They must also explain what is wrong with those that are not valid.

- a. **ch'uti'in lē tinamit** (valid)
- b. **quej lē bak** (not valid because the noun precedes the adjective)
- c. **lē juyub sak** (not valid because lē shouldn't be first)
- d. **bak lē sak** (not valid because there is no noun)
- e. **patzapic lē lol** (valid)
- f. **balam lē quej** (not valid because there is no adjective)

Notice that this process requires students to articulate the Quiche grammar rule they have discovered. The next step is to write the rule. Most students will come up with something like this:

*Rule of Inference QRI.* A Quiche theorem may be constructed by writing an adjective followed by **lē** followed by a noun.

Most will not define “noun” or “adjective”. So after some discussion and prodding they will produce the set-theoretic definitions.

Definition QD1. Each element of the set {**tz'i, che', ja, tinamit, tz'iquin, juyub, chicop, utiw, quej, abaj, lol**} is a noun.

Definition QD2. Each element of the set {*ke'k, sak, cak, rax, nim, co, ch'uti'n, bak, joron*} is an adjective.

With the formal definitions and rule of inference in hand, students can now prove the following theorems.

QT1. **k'ek lē tz'i**

QT2. **sak lē tz'i**

QT3. **nim lē juyub**

Students find the previous examples relatively easy to handle. However, one of the wonderful features of human language is its complexity. So, more advanced problems can be posed. For example, consider these sentences in Comanche, a language of North America (Godby, Wallace and Jolly, 1982).

**paruuku hunuru nohiniyu**

*The raccoon was playing around in the creek.*

**wasape hunuru nohiniyu**

*The bear was playing around in the creek.*

**paruuku paaru nohiniyu**

*The raccoon was playing around in the water.*

**paruuku hunu hunukuhpaiki nohiniyu**

*The raccoon was playing around through the creek.*

**paruuku hunuru nihkanliyu**

*The raccoon was dancing around in the creek.*

**paruuku hunuru nohiyu**

*The raccoon was playing in the creek.*

**wasape hunukuhpaiki sariia miakiiyu**

*The bear was chasing the dog through the creek.*

**sarii kwasinavooa kihtsiaayu**

*The dog was biting the snake.*

It'll take some work, but eventually students working together will be able to devise definitions and rules that allow them to prove this Comanche theorem and translate it into English.

CT1. **sarii hunukuhpaiki paaruku miakiiyu**

The patterns of human languages can serve as useful examples of theorem formation and proof for students. They can reach the point of conjecture and proof with natural languages in a short period of time. Often those students who struggle with algorithmic mathematics can find success in proving language theorems. It is then an easier transition to mathematical proof.

Analyzing language structures has the added feature of providing connections between mathematics and a wide range of disciplines in the humanities and social sciences. Keep in mind that teaching a foreign language is not the goal. Rather, we use the formal structure of a language as an example of an axiomatic system.

## References

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