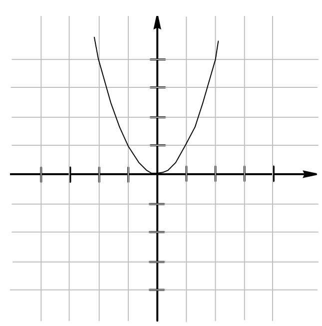
MA 114 - Exam 1 KEY - Spring 2008

________________________________, come on down!

(1 point each) Answer question 1-5 with either TRUE or FALSE.

1. FALSE When labeling the quandrants of the Cartesian plane by number, we label the quadrant where both the x-coordinate and y-coordinate of the points in that quadrant are positive as quadrant I, and then number the other three quadrants by going around the plane clockwise.
2. TRUE If the graph of an equation is symmetric with respect to the x-axis and with respect to the y-axis, then that graph will also be symmetric with respect to the origin.
3. FALSE The slope of a horizontal line is undefined.
4. FALSE The most useful form of an equation of a line is the general form.
5. TRUE, OF COURSE! Functions is my favorite course!
6. (1 point) Find an equation of the vertical line through the point $(-3, 5)$. $\underline{x = -3}$
7. (1 point) If the graph of an equation is symmetric with respect to the origin and the point $(-2, 4)$ is on the graph, what other point must also be on the graph? $\underline{(2, -4)}$
8. (1 point) Find the y-intercept of the line $y = -\frac{1}{2}x + 3$. $\underline{(0, 3)}$
9. (1 point) What is the slope of any line parallel to the line with the equation $y = -\frac{1}{2}x + 3$? $\underline{-\frac{1}{2}}$
10. (1 point) Draw a graph that is symmetric with respect to the y-axis, but is neither symmetric with respect to the x-axis nor with respect to the origin.



11. (10 points) Solve by hand: $\dfrac{2x}{(x+2)(x-2)} = \dfrac{4}{(x+2)(x-2)} - \dfrac{1}{x+2}$.

$$\begin{aligned}
\frac{2x}{(x+2)(x-2)} &= \frac{4}{(x+2)(x-2)} - \frac{1}{x+2} \\
(x+2)(x-2)\left(\frac{2x}{(x+2)(x-2)}\right) &= \left(\frac{4}{(x+2)(x-2)} - \frac{1}{x+2}\right)(x+2)(x-2) \\
2x &= 4 - (x-2) \\
2x &= 6 - x \\
3x &= 6 \\
x &= 2
\end{aligned}$$

However, checking the solution $x = 2$, we see that we cannot substitute $x = 2$ in the original equation. Therefore, this equation has no solution.

12. (10 points) Find all values of y such that the distance from $(2, y)$ to $(-6, 3)$ is 10.

$$\begin{aligned} 10 &= \sqrt{(2-(-6))^2 + (y-3)^2} \\ 100 &= 8^2 + (y-3)^2 \\ 36 &= (y-3)^2 \\ \pm 6 &= y-3 \\ y &= 3 \pm 6 \\ y &= -3, 9 \end{aligned}$$

13. (3 points each) Consider the equation $x + 4y^2 = 9$.

(a) Find the x-intercepts of the graph of this equation, if any.

$$\begin{aligned} x + 4(0)^2 &= 9 \\ x &= 9 \end{aligned}$$

So, the x-intercept is $(9, 0)$.

(b) Find the y-intercepts of the graph of this equation, if any.

$$\begin{aligned} 0 + 4y^2 &= 9 \\ 4y^2 &= 9 \\ y^2 &= \frac{9}{4} \\ y &= \pm\sqrt{\frac{9}{4}} \\ &= \pm\frac{3}{2} \end{aligned}$$

So, the y-intercepts are $\left(0, -\frac{3}{2}\right)$ and $\left(0, \frac{3}{2}\right)$.

(c) Determine if the graph of this equation is symmetric with respect to the x-axis.

$$\begin{aligned} x + 4(-y)^2 &= 9 \\ x + 4y^2 &= 9 \end{aligned}$$

Since this equation is the same as the original, we see that the graph of this equation is symmetric with respect to the x-axis.

(d) Determine if the graph of this equation is symmetric with respect to the y-axis.

$$\begin{aligned} (-x) + 4y^2 &= 9 \\ -x + 4y^2 &= 9 \end{aligned}$$

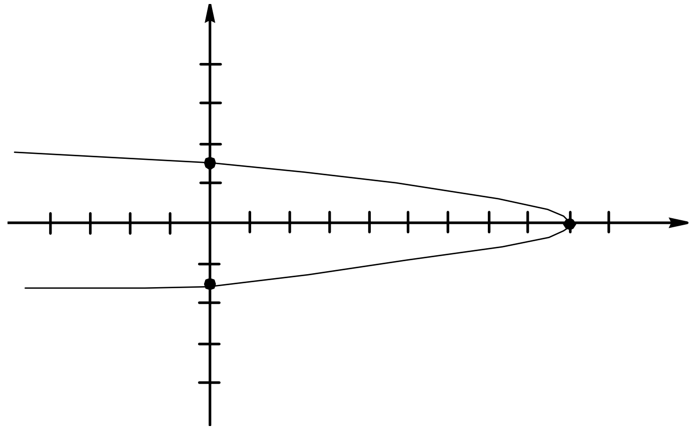
Since this equation is not the same as the original, we see that the graph of this equation is not symmetric with respect to the y-axis.

(e) Determine if the graph of this equation is symmetric with respect to the origin.

$$\begin{aligned} (-x) + 4(-y)^2 &= 9 \\ -x + 4y^2 &= 9 \end{aligned}$$

Since this equation is not the same as the original, we see that the graph of this equation is not symmetric with respect to the origin.

(f) Sketch a graph of this equation using the information you found in parts (a)-(e).



14. Given the points $(-3, 6)$ and $(7, 1)$, find:

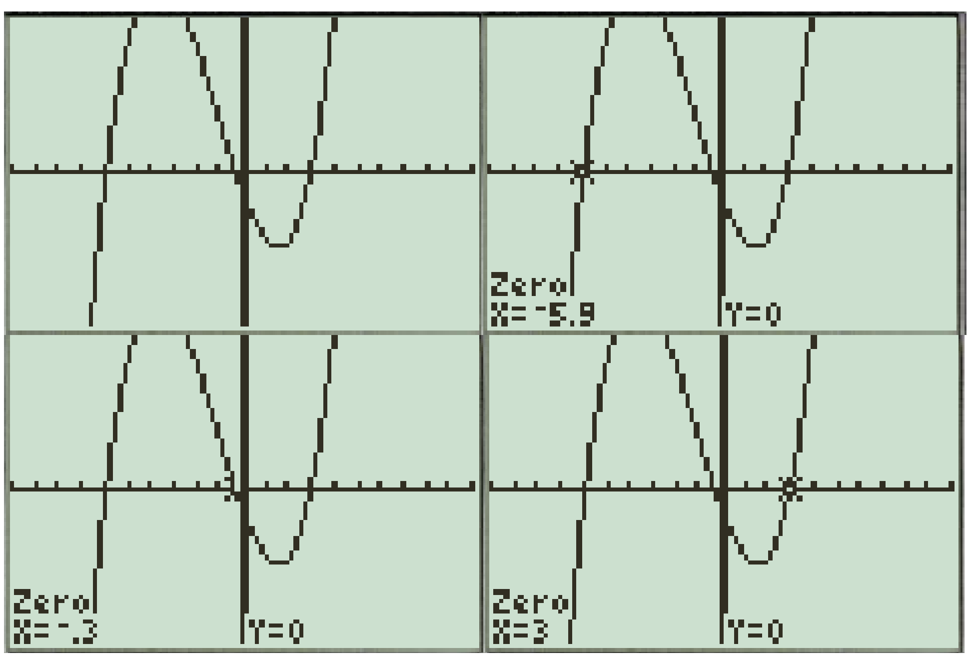
    (a) (6 points) The slope of the line through these points.

$$\begin{aligned} m &= \frac{1-6}{7-(-3)} \\ &= \frac{-5}{10} \\ &= -\frac{1}{2} \end{aligned}$$

    (b) (6 points) An equation of the line perpendicular to this line through $(7, 1)$.

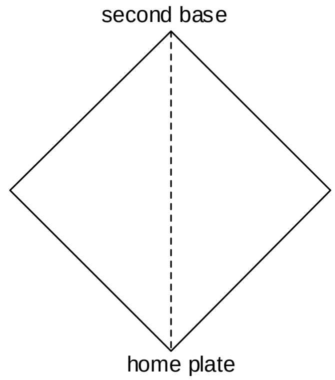
$$y - 1 = 2\,(x - 7)$$

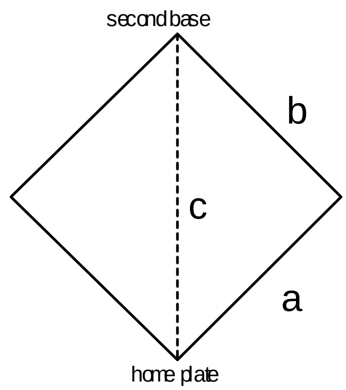
15. (8 points) Use your calculator to find ALL solutions to the equation $x^3+3.2x^2-16.83x-5.31 = 0$. Round your answer to two decimal places.



So, the solutions are $x = -5.9, -0.3, 3$.

16. (8 points) A major league baseball diamond is actually a square, 90 feet on a side. What is the distance directly from home plate to second base?





Using the Pythagorean theorem,

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 90^2 + 90^2 &= c^2 \\ c^2 &= 16200 \\ c &= \sqrt{16200} \\ &= 90\sqrt{2} \text{ feet} \\ &\approx 127.28 \text{ feet} \end{aligned}$$

17. (12 points) Find the center and radius of the circle with the equation $x^2+y^2-8x+12y+3=0$.

$$\begin{aligned} x^2 + y^2 - 8x + 12y + 3 &= 0 \\ x^2 + y^2 - 8x + 12y &= -3 \\ \left(x^2 - 8x + 16\right) + \left(y^2 + 12y + 36\right) &= -3 + 16 + 36 \\ (y-4)^2 + (y+6)^2 &= 49 \end{aligned}$$

So, the center is $(4, -6)$ and the radius is $\sqrt{49} = 7$.

18. (12 points) Find an equation of the circle with endpoints of a diameter of $(-5, 12)$ and $(11, -18)$.

$$\begin{aligned} \text{center} &= \text{midpoint of diameter} \\ &= \left(\frac{-5+11}{2}, \frac{12+(-18)}{2}\right) \\ &= (3, -3) \end{aligned}$$

$$\begin{aligned}
\text{radius} &= \text{distance from center to one of the endpoints} \\
&= \sqrt{(3-(-5))^2 + (-3-12)^2} \\
&= \sqrt{8^2 + (-15)^2} \\
&= \sqrt{64+225} \\
&= \sqrt{289} \\
&= 17
\end{aligned}$$