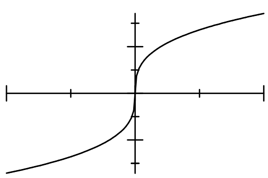
# MA 114 - Exam 2 - Spring 2008

Hello, my name is ________________________________________.
**Directions**: Answer the following questions. Show all work. An answer with no work receives no credit. Saying, "I did it on my calculator" do not constitute work unless the problem specifically asks you to use your calculator.
(1 point each) Answer questions 1-5 with either TRUE or FALSE.

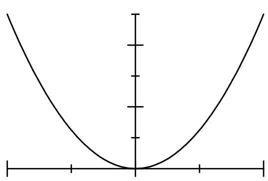
1. TRUE A function may have more than one $x$-intercept.
2. FALSE A function may have more than one $y$-intercept.
3. FALSE The domain of $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, and $\left(\frac{f}{g}\right)(x)$ is the set of all numbers $x$ that are in the domains of both $f(x)$ and $g(x)$.
4. TRUE If $f(x)$ is increasing from $x=a$ to $x=b$, then the average rate of change of $f(x)$ from $x=a$ to $x=b$ is positive.
5. TRUE Today is Monday.

(1 point each) In questions 6-10, match the function with its graph.

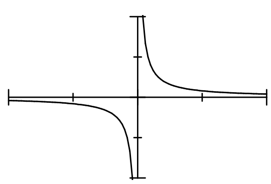
6. C



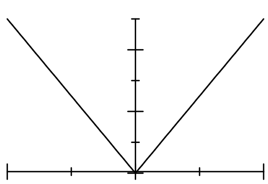
7. E



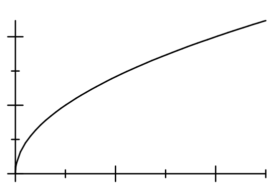
8. H



9. A



10. F



A. $f(x) = |x|$

B. $f(x) = x^3$

C. $f(x) = \sqrt[3]{x}$

D. $f(x) = b$

E. $f(x) = x^2$

F. $f(x) = \sqrt{x}$

G. $f(x) = mx+b$

H. $f(x) = \frac{1}{x}$

11. (3 points each) Determine whether each of the following represents $y$ as a function of $x$. Justify your answers.

(a) $\{(1,3), (2,-1), (3,-2), (4,-1)\}$
This represents a function because the $x$-coordinates of 1, 2, 3, and 4 only appear once each.

(b) $2x^2 + y^2 = 9$

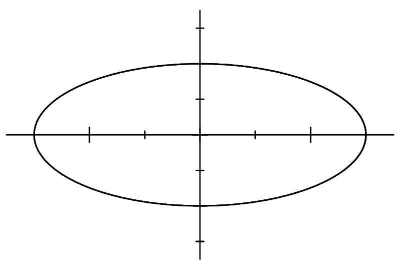
$$\begin{aligned} 2x^2 + y^2 &= 9 \\ y^2 &= 9 - 2x^2 \\ y &= \pm\sqrt{9 - 2x^2} \end{aligned}$$

Since we have plus or minus on the right-hand side, there are two outputs possible for some inputs for $x$. (For instance, plugging in $x = 0$ gives $y = \pm 3$.) So, this equation does not represent a function.

(c) $y + 3 = \sqrt{x - 3}$

$$\begin{aligned} y + 3 &= \sqrt{x - 3} \\ y &= -3 + \sqrt{x - 3} \end{aligned}$$

Since we can solve for $y$ without using plus or minus, we get that each input for $x$ has only one output. So, this is a function.

(d) 

This is not a function since the graph does not pass the vertical line test.

12. For the function $f(x) = \dfrac{x}{x^2 - 1}$,

(a) (2 points) Find the $y$-intercept.

$$\begin{aligned} f(0) &= \frac{0}{0^2 - 1} \\ &= 0 \end{aligned}$$

So, the $y$-intercept is $(0, 0)$.

(b) (6 points) Find the domain of $f(x)$. Express your answer in interval notation.
Setting the denominator equal to zero and solving, we have

$$\begin{aligned} x^2 - 1 &= 0 \\ (x+1)(x-1) &= 0 \\ x = -1 \quad &\text{or} \quad x = 1 \end{aligned}$$

These are the values that cannot be plugged in for $x$. So, the domain of $f(x)$ is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

(c) (2 points) Determine whether $f(x)$ is even, odd, or neither, justifying your answer.

$$\begin{aligned} f(-x) &= \frac{-x}{(-x)^2-1} \\ &= -\frac{x}{x^2-1} \\ &= -f(x) \end{aligned}$$

So, $f(x)$ is an odd function.

13. Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{4-2x}$.

(a) (2 points) Find $\left(\frac{f}{g}\right)(x)$.

$$\begin{aligned} \left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{x^2-4}{\sqrt{4-2x}} \end{aligned}$$

(b) (6 points) Find the domain of $\left(\frac{f}{g}\right)(x)$. Express your answer in interval notation.

We cannot divide by zero, and we cannot take the square root of a negative number. So, we need

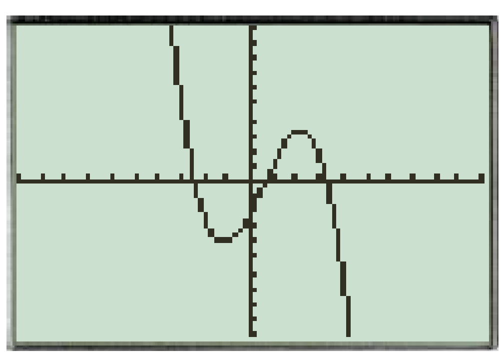
$$\begin{aligned} 4-2x &> 0 \\ -2x &> -4 \\ x &< 2 \end{aligned}$$

So, the domain of $\left(\frac{f}{g}\right)(x)$ is $(-\infty, 2)$.

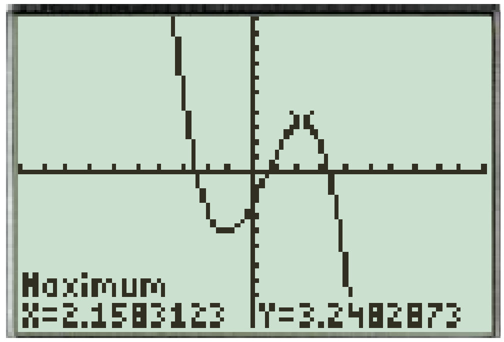
14. Consider the function $f(x) = -0.4x^3 + 0.6x^2 + 3x - 2$.

(a) (4 points) Using your calculator, find the coordinates of the points at which $f(x)$ has local maxima and local minima. Be sure to label whether each point is a local maximum or a local minimum. Round your answers to two decimal places. Explain how you used your calculator to get your solution.

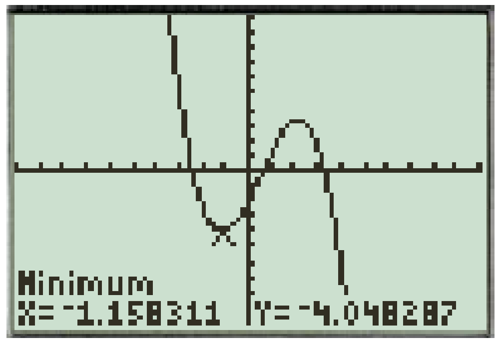
Graphing $y = f(x)$, we see



There is a local maximum near $x = 2$ and a local minimum near $x = -1$. Using the calculator, we see




So, $f(x)$ has a local maximum at $(2.16, 3.25)$. We also have




So, $f(x)$ has a local minimum at $(-1.16, -4.05)$.

(b) (2 points) Determine the intervals on which $f(x)$ is increasing. State your answer in interval notation.

From the graph, we see $f(x)$ is increasing on $(-1.16, 2.16)$.

(c) (2 points) Determine the intervals on which $f(x)$ is decreasing. State your answer in interval notation.

From the graph, we see $f(x)$ is decreasing on $(-\infty, -1.16) \cup (2.16, \infty)$.

15. Suppose that a company has just purchased a new truck for \$30,000. The company chooses to depreciate the truck using the straight-line method over 15 years. (This means the truck will be worth \$0 in 15 years, and it loses the same amount of value each year.)

(a) (8 points) Write a function that expresses the value of the truck as a function of its age.

Let $x$ be the age of the truck and $y$ be the value of the truck at age $x$.

When $x = 0$, $y = 30000$. When $x = 15$, $y = 0$. So, the slope of the line is

$$\frac{0 - 30000}{15 - 0} = -2000$$

The $y$-intercept is $(0, 30000)$. So, the function would be

$$y = -2000x + 30000$$

(b) (4 points) Using your function from part (a), what is the value of the truck when it is 7 years old?

When $x = 7$, we have

$$\begin{aligned} y &= -2000(7) + 30000 \\ &= 16000 \end{aligned}$$

So, the truck will be worth \$16,000 when it is 7 years old.

(c) (4 points) Using your function from part (a), when will the truck be worth \$8,000?
We want $y = 8000$. So, we have

$$\begin{aligned} 8000 &= -2000x + 30000 \\ -22000 &= -2000x \\ x &= 11 \end{aligned}$$

So, in 11 years, the truck will be worth \$8,000.

16. (10 points) The gravitational force $F$ on an object varies directly with the mass $m$ of the object. If an object with mass of 10 has a gravitational force on it of 98, what gravitational force would an object with mass 32 have on it?

$$\begin{aligned} F &= km \\ 98 &= k(10) \\ k &= 9.8 \end{aligned}$$

$$\begin{aligned} F &= 9.8m \\ F &= 9.8(32) \\ &= 313.6 \end{aligned}$$

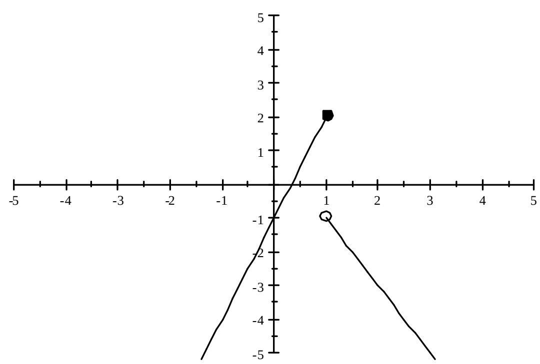
17. Consider $f(x) = \begin{cases} 3x - 1 & \text{if } x \leq 1 \\ -2x + 1 & \text{if } x > 1 \end{cases}$.

(a) (2 points) Find $f(2)$.
$f(2) = -2(2) + 1 = -3$

(b) (2 points) Find $f(0)$.
$f(0) = 3(0) - 1 = -1$

(c) (2 points) Find $f(1)$.
$f(1) = 3(1) - 1 = 2$

(d) (6 points) Sketch a graph of $y = f(x)$.




18. For the function $g(x) = x^2 - x - 2$,

(a) (4 points) Find the $x$-intercepts.

$$\begin{aligned} g(x) &= 0 \\ x^2 - x - 2 &= 0 \\ (x+1)(x-2) &= 0 \\ x = -1 \quad &\text{or} \quad x = 2 \end{aligned}$$

So, the $x$-intercepts are $(-1, 0)$ and $(2, 0)$.

(b) (4 points) Find the $x$-values at which $g(x) = 4$.

$$\begin{aligned} g(x) &= 4 \\ x^2 - x - 2 &= 4 \\ x^2 - x - 6 &= 0 \\ (x+2)(x-3) &= 0 \\ x &= -2, 3 \end{aligned}$$

(c) (6 points) Find the difference quotient of $g(x)$, that is, find $\frac{g(x+h) - g(x)}{h}$, $h \neq 0$, and simplify.

$$\begin{aligned} \frac{g(x+h) - g(x)}{h} &= \frac{\left((x+h)^2 - (x+h) - 2\right) - \left(x^2 - x - 2\right)}{h} \\ &= \frac{x^2 + 2xh + h^2 - x - h - 2 - x^2 + x + 2}{h} \\ &= \frac{2xh + h^2 - h}{h} \\ &= \frac{h(2x + h - 1)}{h} \\ &= 2x + h - 1 \end{aligned}$$