

MA 114 - Exam 3 - Spring 2008

And the next American Idol is _____!

Directions: Answer the following questions. Show all work. An answer with no work receives no credit. Saying, "I did it on my calculator" does not constitute work unless the problem specifically asks you to use your calculator.

(1 point each) Answer questions 1-5 with either TRUE or FALSE.

1. TRUE Except for $f(x) = 0$, there are no functions that are symmetric with respect to the x -axis.
2. FALSE If $f(x) = ax^2 + bx + c$ and a is negative, then the vertex of the parabola represents the minimum of $f(x)$.
3. FALSE If $f(x)$ is a polynomial of degree n , then $f(x)$ has at most n turning points.
4. TRUE If $f(x)$ is a polynomial of degree n , then $f(x)$ has at most n real zeros.
5. TRUE Today is Tuesday.

(1 point each) In questions 6-10, give an example of a function that has the property or properties described. Do not simply sketch a graph - write out the rule for the function. (In some cases, there are many possible answers.)

6. Write the rule for the function whose graph is the reflection of the graph of $y = \sqrt{x}$ about the y -axis.

$$y = \sqrt{-x}$$

7. Give an example of a quadratic function with a graph that has an axis of symmetry of $x = -3$.

$$y = (x + 3)^2$$

8. Give an example of a polynomial function $f(x)$ with the following end behavior.

$$\text{As } x \rightarrow \infty, f(x) \rightarrow -\infty$$

$$\text{As } x \rightarrow -\infty, f(x) \rightarrow \infty$$

$$f(x) = -x$$

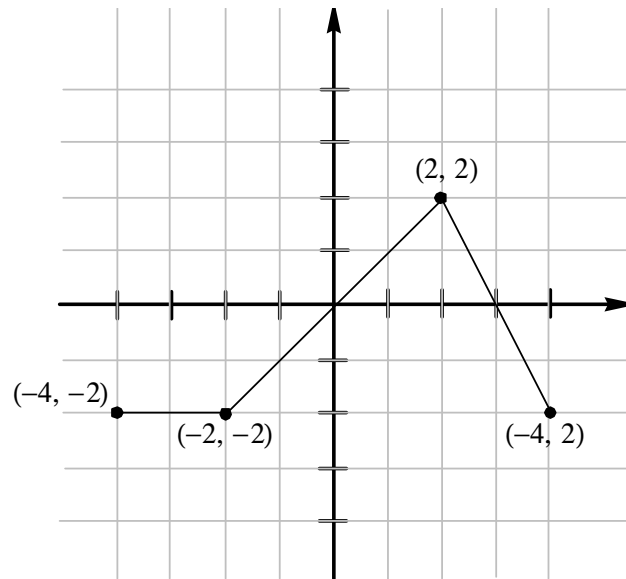
9. Give an example of a rational function $f(x)$ that has a horizontal asymptote of $y = 0$.

$$f(x) = \frac{1}{x}$$

10. Give an example of a polynomial function $f(x)$ that has degree 4 and has exactly three distinct zeros.

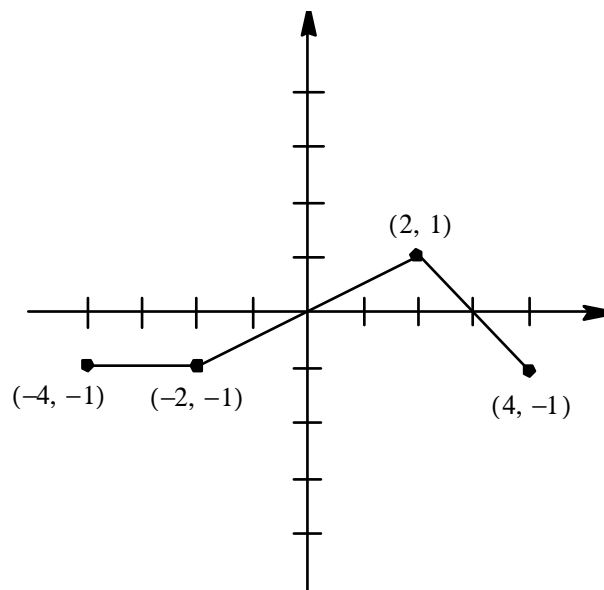
$$f(x) = x^2(x - 1)(x - 2)$$

11. (10 points) The graph of $y = f(x)$ is given below. List the transformations necessary to sketch a graph of $H(x) = \frac{1}{2}f(x + 1) - 2$. Using these transformations, sketch a graph of $H(x)$, showing all the steps.

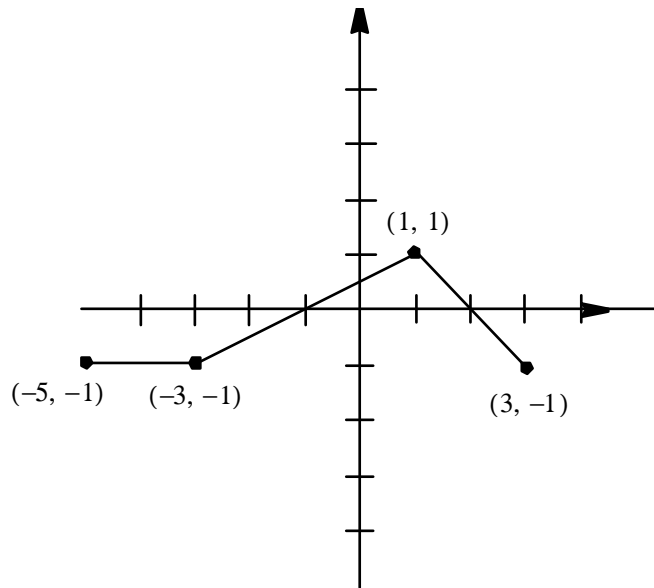


Graph of $y = f(x)$

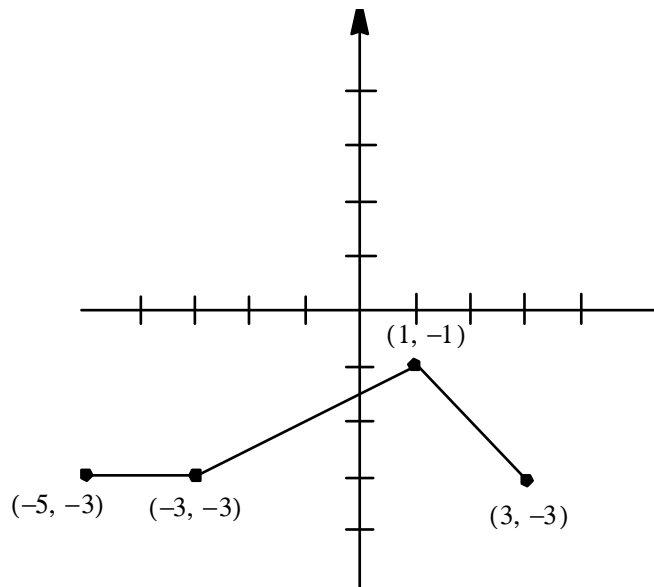
1. Multiply all y -coordinates by $\frac{1}{2}$.
2. Shift left 1 unit.
3. Shift down 2 units.



After first transformation



After second transformation



Final graph of $H(x)$

12. (6 points) The John Deere company has found that the revenue from sales of heavy-duty tractors is a function of the unit price p (in dollars) that it charges. If the revenue R is $R(p) = -\frac{1}{2}p^2 + 1900p$, what unit price p (in dollars) should be charged to maximize revenue? What is the maximum revenue?

$$\begin{aligned} -\frac{b}{2a} &= -\frac{1900}{2\left(-\frac{1}{2}\right)} \\ &= 1900 \end{aligned}$$

So, the company should charge \$1900 for the tractors in order to maximize revenue. The maximum revenue is

$$\begin{aligned} R(1900) &= -\frac{1}{2}(1900)^2 + 1900(1900) \\ &= \$1,805,000 \end{aligned}$$

13. (10 points) The volume of a gas V held at a constant temperature in a closed container varies inversely with its pressure F . If the volume of a gas is 600 cubic centimeters (cm^3) when the pressure is 150 millimeters of mercury (mm Hg), find the volume when the pressure is 200 mm Hg.

$$\begin{aligned} V &= \frac{k}{F} \\ 600 &= \frac{k}{150} \\ k &= 600(150) \\ &= 90000 \end{aligned}$$

$$\begin{aligned} V &= \frac{90000}{F} \\ V &= \frac{90000}{200} \\ &= 450 \text{ cm}^3 \end{aligned}$$

14. For the polynomial $f(x) = -3(x+4)^2(x-1)^3(x+3)$, determine

- (a) (2 points) the degree of the polynomial

The degree of the polynomial is 6.

- (b) (2 points) the power function $f(x)$ resembles as $|x|$ gets large, $f(x)$ resembles $-3x^6$ as $|x|$ gets large.
- (c) (4 points) the x - and y -intercepts of the graph of $f(x)$,

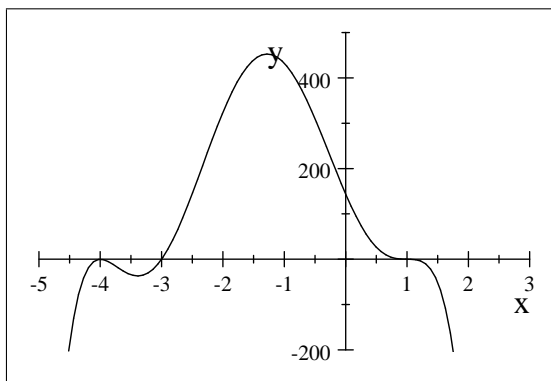
$$\begin{aligned} f(x) &= 0 \\ -3(x+4)^2(x-1)^3(x+3) &= 0 \\ x &= -4, 1, -3 \end{aligned}$$

So, the x -intercepts are $(-4, 0)$, $(-3, 0)$, and $(1, 0)$.

$$\begin{aligned} f(0) &= -3(0+4)^2(0-1)^3(0+3) \\ &= -3(16)(-1)(3) \\ &= 144 \end{aligned}$$

So, the y -intercept is $(0, 144)$.

- (d) (2 points) whether the graph of $f(x)$ crosses or touches the x -axis at each x -intercept,
 At $x = -4$, the graph touches the x -axis.
 At $x = -3$, the graph crosses the x -axis.
 At $x = 1$, the graph crosses the x -axis.
- (e) (4 points) the coordinates of any turning points using your calculator, correct to two decimal places, and
 From the graph of $y = f(x)$, we see $f(x)$ has three turning points: $(-4, 0)$, $(-3.39, -36, 83)$, and $(-1.28, 452.47)$.
- (f) (4 points) sketch a graph of $f(x)$.



15. Let $f(x) = \frac{x^2 - 7x + 10}{x^2 - x - 12}$, find

(a) (3 points) the domain,

$$f(x) = \frac{(x-2)(x-5)}{(x-4)(x+3)}$$

So, x cannot equal 4 or -3 . So, the domain is $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$.

(b) (3 points) the x - and y -intercepts,

$$\begin{aligned} f(x) &= 0 \\ \frac{(x-2)(x-5)}{(x-4)(x+3)} &= 0 \\ (x-2)(x-5) &= 0 \\ x &= 2, 5 \end{aligned}$$

So, the x -intercepts are $(2, 0)$ and $(5, 0)$.

$$\begin{aligned} f(0) &= \frac{0^2 - 7(0) + 10}{0^2 - 0 - 12} \\ &= -\frac{5}{6} \end{aligned}$$

So, the y -intercept is $(0, -\frac{5}{6})$.

(c) (2 points) the vertical asymptotes (if any),

The denominator equals zero when $(x-4)(x+3) = 0$, or when $x = 4, -3$. So, the vertical asymptotes are $x = 4$ and $x = -3$.

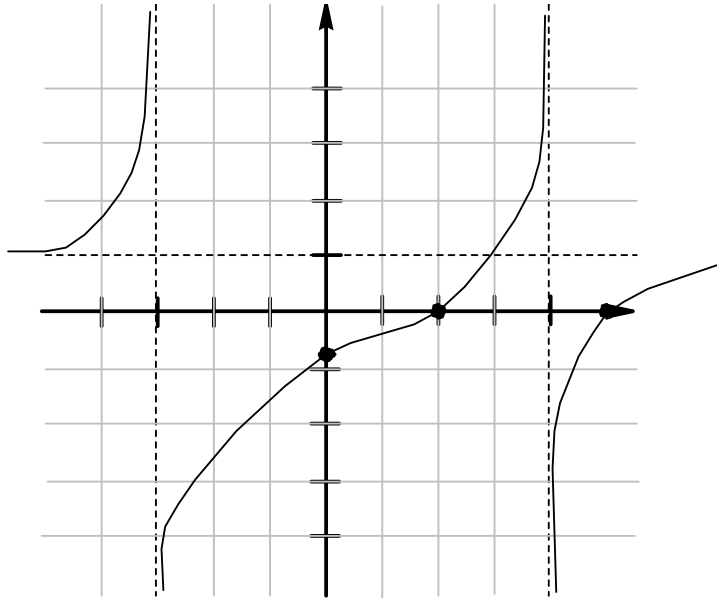
(d) (2 points) the horizontal asymptotes (if any),

Since the degrees of the top and bottom are equal, we can find the horizontal asymptote by looking at the ratio of the leading coefficients. So, the horizontal asymptote is $y = \frac{1}{1} = 1$.

(e) (2 points) the oblique asymptotes (if any), and

Since the degree of the top is not exactly one larger than the degree of the bottom, this function has no oblique asymptote.

(f) (4 points) sketch a graph of $f(x)$, showing all the above information on your graph.

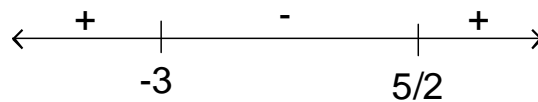


16. Solve the following inequalities.

(a) (8 points) $-x < 2x^2 - 15$

$$\begin{aligned} -x &< 2x^2 - 15 \\ 0 &< 2x^2 + x - 15 \\ 0 &< (2x - 5)(x + 3) \end{aligned}$$

We see $(2x - 5)(x + 3) = 0$ when $x = \frac{5}{2}, -3$. Making a sign chart, we see

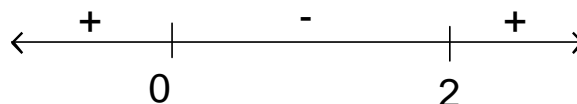


We are looking for when the expression is positive. So, the solution is $(-\infty, -3) \cup (\frac{5}{2}, \infty)$.

(b) (8 points) $\frac{x^3 - x^2 - 2x}{x + 1} \leq 0$

$$\begin{aligned} \frac{x^3 - x^2 - 2x}{x + 1} &\leq 0 \\ \frac{x(x^2 - x - 2)}{x + 1} &\leq 0 \\ \frac{x(x - 2)(x + 1)}{x + 1} &\leq 0 \\ x(x - 2) &\leq 0, \quad x \neq -1 \end{aligned}$$

We see $x(x - 2) = 0$ when $x = 0, 2$. Making a sign chart, we have



We are looking for when the expression is negative. So, the solution is $[0, 2]$.

17. Let $f(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$.

(a) (4 points) List all the possible rational zeros of $f(x)$.

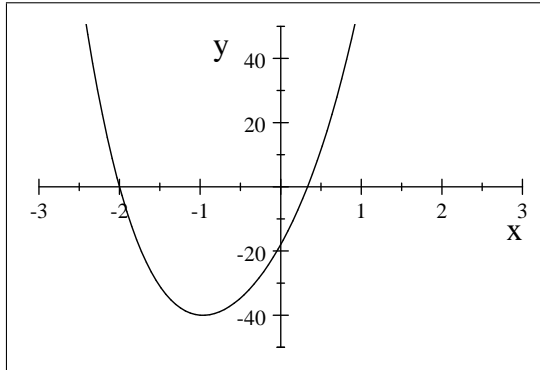
Factors of 18 are $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$

Factors of 3 are $\pm 1, \pm 3$

Possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm \frac{1}{3}, \pm \frac{2}{3}$

(b) (6 points) Find the real zeros of $f(x)$.

Here is a graph of $f(x)$.



It appears that there is a zero at $x = -2$.

$$\begin{array}{r|rrrrr} -2 & 3 & 5 & 25 & 45 & -18 \\ & & -6 & 2 & -54 & 18 \\ \hline & 3 & -1 & 27 & -9 & 0 \end{array}$$

So, $x = -2$ is a zero. It also appears that there is a zero at $x = \frac{1}{3}$.

$$\begin{array}{r|rrrrr} -2 & 3 & 5 & 25 & 45 & -18 \\ & & -6 & 2 & -54 & 18 \\ \hline 1/3 & 3 & -1 & 27 & -9 & 0 \\ & & 1 & 0 & 9 & \\ \hline & 3 & 0 & 27 & 0 & \end{array}$$

So, $x = \frac{1}{3}$ is a zero. The remaining polynomial is $3x^2 + 27$. We see this has no real zeros. So, the real zeros of $f(x)$ are $x = -2, \frac{1}{3}$.

(c) (4 points) Write $f(x)$ in factored form.

$$\begin{aligned} f(x) &= (x + 2) \left(x - \frac{1}{3} \right) (3x^2 + 27) \\ &= 3(x + 2) \left(x - \frac{1}{3} \right) (x^2 + 9) \end{aligned}$$