# MA 125 - In-Class Assignment 1 - Spring 2008

Hello, my name is _____

1. Five hundred students at a university are voting for a new school mascot. After 300 votes have been counted, the tallies are as follows.

| | |
|---|---|
| Purple People Eaters | 106 |
| Flying Squirrels | 98 |
| Rambling Platypi | 96 |

   (a) What is the minimum number of the remaining votes needed to guarantee that the new mascot will be the Purple People Eaters? Explain.

   The closest competitor is Flying Squirrels. To tie, Flying Squirrels would need 8 votes, leaving $200 - 8 = 192$ votes remaining. Purple People Eaters would need a majority of these votes. Since $192 \div 2 = 96$, Purple People Eaters would need 97 of the remaining votes to guarantee victory.

   (b) What is the minimum number of the remaining votes needed to guarantee that the new mascot will be the Flying Squirrels? Explain.

   To tie Purple People Eaters, Flying Squirrels would need 8 votes, leaving $200 - 8 = 192$ votes remaining. Flying Squirrels would need a majority of these votes. Since $192 \div 2 = 96$, Flying Squirrels would need 97 of these remaining votes, plus the additional 8 to tie, to guarantee victory. So, Flying Squirrels would need $97 + 8 = 105$ votes.

   (c) What is the minimum number of the remaining votes needed to guarantee that the new mascot will be the Rambling Platypi? Explain.

   To tie Purple People Eaters, Rambling Platypi would need 10 votes, leaving $200 - 10 = 190$ votes remaining. Flying Squirrels would need a majority of these votes. Since $190 \div 2 = 95$, Flying Squirrels would need 96 of these remaining votes, plus the additional 10 to tie, to guarantee victory. So, Flying Squirrels would need $96 + 10 = 106$ votes.

2. Suppose there were 15,364 votes an election involving six candidates.

   (a) If a candidate is required to have majority of the votes cast to be considered the winner, what is the minimum number of votes needed to win? Explain.

   A majority win requires a vote tally over 50%. Fifty percent of $15,364$ is $15364 \times .5 = 7682$. So, a candidate would need 7683 votes to have over 50% and thus have a majority.

   (b) If a candidate is needs a plurality of the votes cast to be considered the winner, what is the minimum number of votes a winning candidate can have and still win the election? Explain.

   Dividing the votes equally among the candidates, we have $15364 \div 6 = 2560.67$. So, each candidate could get 2560 votes, leaving four remaining votes. One extra vote for a candidate will not gain that candidate victory as the remaining three votes must go elsewhere, giving another candidate a total of at least 2561. So, a candidate must have an addition two of the four votes, or 2562, in order to win. (This does not guarantee victory, however.)

3. A university has 75 faculty members, and they need to vote for a faculty representative to the Board of Trustees. There are four candidates running for this position, and the preference

rankings are as follows.

| | Number of Voters | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **10** | **8** | **9** | **4** | **14** | **6** | **11** | **13** |
| Adkins | 1 | 1 | 3 | 2 | 2 | 3 | 2 | 2 |
| Blythe | 2 | 4 | 1 | 1 | 3 | 2 | 3 | 4 |
| Cassidy | 3 | 2 | 4 | 3 | 1 | 1 | 4 | 3 |
| Dearborn | 4 | 3 | 2 | 4 | 4 | 4 | 1 | 1 |

(a) Which candidate wins a plurality election?

$$\begin{aligned} \text{Adkins} &= 18 \\ \text{Blythe} &= 13 \\ \text{Cassidy} &= 20 \\ \text{Dearborn} &= 24 \end{aligned}$$

So, Dearborn wins.

(b) Which candidate wins a plurality election with a runoff between the top two finishers?

From part (a), we see the runoff would be between Dearborn and Cassidy. All of those who voted for Adkins and four of those who voted for Blythe prefer Cassidy to Dearborn, giving Cassidy $20 + 18 + 4 = 42$ votes, while nine of those who voted for Blythe prefer Dearborn to Cassidy, giving Dearborn $24 + 9 = 33$ votes. So, Cassidy wins.

(c) Which candidate wins a plurality election with a runoff between the top three finishers, and then a runoff between the top two finishers of the first runoff?

From part (a), the top three finishers are Adkins, Cassidy, and Dearborn. So, in the runoff among these three, four of the Blythe voters would vote for Adkins, and nine would vote for Dearborn. This makes the new vote count

$$\begin{aligned} \text{Adkins} &= 18 + 4 = 22 \\ \text{Cassidy} &= 20 \\ \text{Dearborn} &= 24 + 9 = 33 \end{aligned}$$

So, now in a runoff between Adkins and Cassidy, the Blythe votes would be split the same way as above, and all of the Cassidy votes would go to Adkins. So, we have

$$\begin{aligned} \text{Adkins} &= 22 + 20 = 42 \\ \text{Dearborn} &= 33 \end{aligned}$$

So, Adkins wins.

(d) In a plurality election, could those who ranked Adkins as their top candidate have achieved a preferable outcome by voting strategically if the others voted as shown in the table? Explain.

The ten Adkins voters who ranked Blythe as their second choice cannot get Blythe elected because Blythe is eleven votes behind Dearborn. However, the eight voters who prefer Cassidy second could throw their support to Cassidy, giving Cassidy enough votes to win. So, yes, those who ranked Adkins as their top candidate can achieve a preferable outcome, namely a victory for Cassidy. (Notice those who ranked Blythe second would also prefer this outcome to Dearborn winning.)

(e) In a plurality election with a runoff between the top two finishers, could those who ranked Blythe first and Dearborn second have achieved a preferable outcome by voting strategically if the others voted as shown in the table? Explain.

In the runoff, the nine who voted for Blythe would vote for Dearborn. So, the outcome of the runoff would not be affected. However, if in the initial plurality election these

voters voted for Adkins, then Adkins would have 27 votes, edging out Cassidy to get in the runoff. Then in that runoff, Adkins would win. So, yes, those who ranked Blythe first and Dearborn second can achieve a preferable outcome of Adkins over Dearborn.

| | Number of Voters | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **10** | **8** | **9** | **4** | **14** | **6** | **11** | **13** |
| Adkins | 1 | 1 | 3 | 2 | 2 | 3 | 2 | 2 |
| Blythe | 2 | 4 | 1 | 1 | 3 | 2 | 3 | 4 |
| Cassidy | 3 | 2 | 4 | 3 | 1 | 1 | 4 | 3 |
| Dearborn | 4 | 3 | 2 | 4 | 4 | 4 | 1 | 1 |

(f) If the voting is sequential, with Blythe and Cassidy squaring off first, then the winner going against Dearborn, and then that winner going against Adkins, who wins?

$$\begin{aligned} \text{Blythe} &= 10+9+4+11=34 \\ \text{Cassidy} &= 8+14+6+13=41 \end{aligned}$$

So, Cassidy wins this pairing. Then

$$\begin{aligned} \text{Cassidy} &= 10+8+4+14+6=42 \\ \text{Dearborn} &= 9+11+13=33 \end{aligned}$$

So, Cassidy wins this pairing. Then

$$\begin{aligned} \text{Adkins} &= 10+8+9+4+11+13=55 \\ \text{Cassidy} &= 14+6=20 \end{aligned}$$

So, Adkins wins.

(g) If the voting is done tournament style, with Adkins and Cassidy competing in one election, and Blythe and Dearborn in the other, and the winners of these elections then running against each other, who wins?

$$\begin{aligned} \text{Adkins} &= 10+8+9+4+11+13=55 \\ \text{Cassidy} &= 14+6=20 \end{aligned}$$

So, Adkins wins this pairing.

$$\begin{aligned} \text{Blythe} &= 10+9+4+14+6=43 \\ \text{Dearborn} &= 8+11+13=32 \end{aligned}$$

So, Blythe wins this pairing.

$$\begin{aligned} \text{Adkins} &= 10+8+14+11+13=56 \\ \text{Blythe} &= 9+4+6=19 \end{aligned}$$

So, Adkins wins.

(h) If Borda's method is used, who wins?

$$\begin{aligned} \text{Adkins} &= 18\times4+42\times3+15\times2+0\times1=228 \\ \text{Blythe} &= 13\times4+16\times3+25\times2+21\times1=171 \\ \text{Cassidy} &= 20\times4+8\times3+27\times2+20\times1=178 \\ \text{Dearborn} &= 24\times4+9\times3+8\times2+34\times1=173 \end{aligned}$$

So, Adkins wins.

(i) If Borda's method is used, could the voters who ranked Blythe first and Dearborn second have achieved a preferable outcome by voting strategically if the others voted as shown in the table? Explain.

Since Adkins wins and is the third preference to Blythe, the only other option to have a preferable result is for these Blythe supporters to vote for Dearborn instead to try to get Dearborn to win (since Blythe did not). To do this most efficiently, they would need to rank Dearborn first and Adkins last. This would change the Borda counts as follows.

$$\begin{aligned} \text{Adkins} &= 18 \times 4 + 42 \times 3 + 6 \times 2 + 9 \times 1 = 219 \\ \text{Dearborn} &= 33 \times 4 + 0 \times 3 + 8 \times 2 + 34 \times 1 = 182 \end{aligned}$$

Adkins would still win in this election. So, no, the voters who ranked Blythe first and Dearborn second cannot achieve a preferable outcome to Adkins winning.