

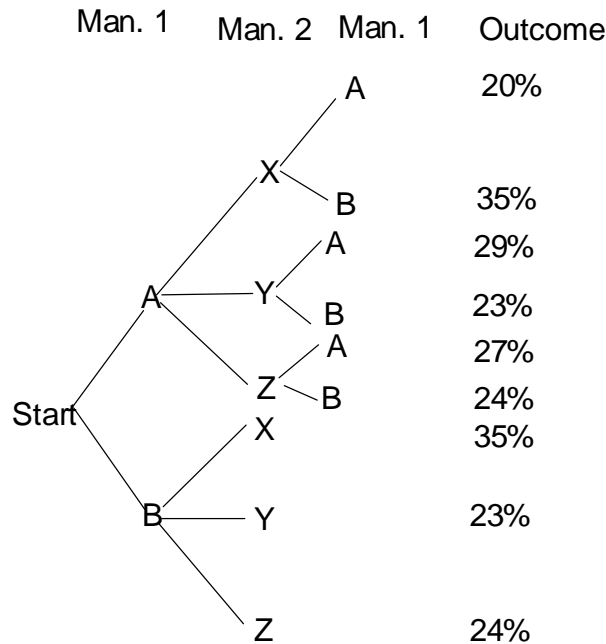
MA 125 - In-Class Assignment 2 Key - Spring 2008

The name's _____, but my friends call me _____.

- For a particular at bat in a baseball game, we define success as the batter get a hit and failure if the batter does not get a hit. Imagine the following scenario. The manager of the team at bat can let Player A hit, or substitute Player B for Player A. The manager of the team in the field can then choose to stay with Pitcher X, or substitute either Pitcher Y or Pitcher Z for Pitcher X. Finally, if Player A was not originally replaced, the manager of the team at bat can again let Player A hit, or substitute Player B for Player A. The success rates (as percentages) of the hitters against the three pitchers are given in the table below.

	Pitcher X	Pitcher Y	Pitcher Z
Player A	20%	29%	27%
Player B	35%	23%	24%

- Construct a game tree for the managers' choices.



- Determine the strategies each manager should follow, assuming best play by both managers.

If Manager 1 opts to go with Player B at the beginning, Manager 2 will bring in Pitcher Y, and the batter will be successful only 23% of the time.

If Manager 1 stays with Player A, then if Manager 2 stays with Pitcher X, Manager 1 will bring in Player B be successful 35% of the time. If Manager 2 brings in Pitcher Y, then Manager 1 will stay with Player A and be successful 29% of the time. If Manager 2 brings in Pitcher Z, Manager 1 will stay with Player A and be successful 27% of the time.

So, Manager 1 will not switch to Player B immediately because the outcomes for staying with Player A at the start are better. Given that Manager 1 will stay with Player A, Manager 2 will switch to Pitcher Z because this gives the lowest percentage of success to the other team. Finally, Manager 1 will stick with Player A to face Pitcher Z.

2. Two rock hounds each have one chunk of meteor. If the first rock hound splits her chunk into 4 pieces and the second splits her chunk into 4 pieces as well, each piece can be sold for \$25. Each rock hound can split her chunk into as many as three additional pieces without the other knowing (until the go to sell the pieces). However, for each additional piece above the 8 total pieces, the price per piece at which each can be sold for decreases by \$3.

- (a) Construct the payoff matrix that gives the profits for each rock hound for all the possibilities.

		Rock Hound 2			
		4	5	6	7
Rock Hound 1	4	(100, 100)	(88, 110)	(76, 114)	(64, 112)
	5	(110, 88)	(95, 95)	(80, 96)	(65, 91)
	6	(114, 76)	(96, 80)	(78, 78)	(60, 70)
	7	(112, 64)	(91, 65)	(70, 60)	(49, 49)

- (b) Find all the equilibrium points of this matrix.

The equilibrium points are (80, 96) and (96, 80).

- (c) Find the reduced payoff matrix.

		Rock Hound 2	
		5	6
Rock Hound 1	5	(95, 95)	(80, 96)
	6	(96, 80)	(78, 78)

- (d) Describe the best strategies for each rock hound.

The best strategy for each rock hound is to select to make either 5 or 6 pieces.

3. Aaron and Alex engage in a pistol duel to the death. Each has one shot. Aaron may fire when he is 60 paces, 40 paces, or 20 paces away, while Alex may fire when he is 50 paces, 30 paces, or 10 paces away. Assume that, if one of the duelists fires and misses, the other one knows this and will walk up and kill the person who missed. Below are the percentages of success for each duelist when firing at the various number of paces.

Aaron		Alex	
Strategy	Percentage of Success	Strategy	Percentage of Success
60 paces	20%	50 paces	25%
40 paces	40%	30 paces	35%
20 paces	70%	10 paces	75%

We can view the duel as a zero-sum game, with the payoffs computed as follows. Decide who will fire first and compute that person's payoff. For example, if Aaron choose to fire at 60 paces, then Aaron shoots first, no matter what. He wins 20% of the time and loses 80% of the time, meaning his payoff will be $20\% - 80\% = -60\%$.

- (a) Find the payoff matrix, with Aaron as the row player.

		Alex		
		50	30	10
Aaron	60	-60%	-60%	-60%
	40	50%	-20%	-20%
	20	50%	30%	40%

(b) Find the reduced payoff matrix.

		Alex
		30
Aaron	20	$\sqrt{30\%}$

(c) Explain both players' optimal strategies.

Alex should select to fire at 30 paces (which means he will fire first).

(d) What is the percentage of time Alex will be killed, assuming best play by both players?

65%