

MA 125 - Quiz 1 - Spring 2008

Call me _____

Directions: Please show all your work leading to your answers. An unsupported answer will receive no credit.

1. Four candidates are running for president of a mathematical society. There are 21 voters in the election, and their preferences and approval rankings are indicated in the table below.

	Number of Voters					
	4	2	1	6	3	5
Barnaby	1√	3	4	4	4	4
Dennis	2√	1√	1√	1√	3	3√
Haggerty	3	2	2√	2√	1√	2√
Rutland	4	4	3√	3	2√	1√

- (a) Which candidate wins a plurality election?

$$\begin{aligned} \text{Barnaby} &= 4 \\ \text{Dennis} &= 9 \\ \text{Haggerty} &= 3 \\ \text{Rutland} &= 5 \end{aligned}$$

So, Dennis wins a plurality election.

- (b) Could those who ranked Haggerty first have achieved a preferable outcome in a plurality election by voting strategically if the others voted as shown in the table? Explain.

Those who ranked first prefer Rutland to Dennis. If these three voters switch to Rutland, the results would be

$$\begin{aligned} \text{Barnaby} &= 4 \\ \text{Dennis} &= 9 \\ \text{Haggerty} &= 0 \\ \text{Rutland} &= 8 \end{aligned}$$

which still is not enough votes for Rutland to win. So, no, those who ranked Haggerty first cannot achieve a preferable outcome in a plurality election by voting strategically if the others voted as shown in the table

- (c) Which candidate wins a plurality election with a runoff between the top two finishers?

The runoff is between Dennis and Rutland. The results of this election are

$$\begin{aligned} \text{Dennis} &= 13 \\ \text{Rutland} &= 8 \end{aligned}$$

So, Dennis wins.

- (d) Which candidate wins an election using Borda's method?

$$\begin{aligned} \text{Barnaby} &= 4 \times 4 + 0 \times 3 + 2 \times 2 + 15 \times 1 = 35 \\ \text{Dennis} &= 9 \times 4 + 4 \times 3 + 8 \times 2 + 0 \times 1 = 64 \\ \text{Haggerty} &= 3 \times 4 + 14 \times 3 + 4 \times 2 + 0 \times 1 = 62 \\ \text{Rutland} &= 5 \times 4 + 3 \times 3 + 7 \times 2 + 6 \times 1 = 49 \end{aligned}$$

So, Dennis wins.

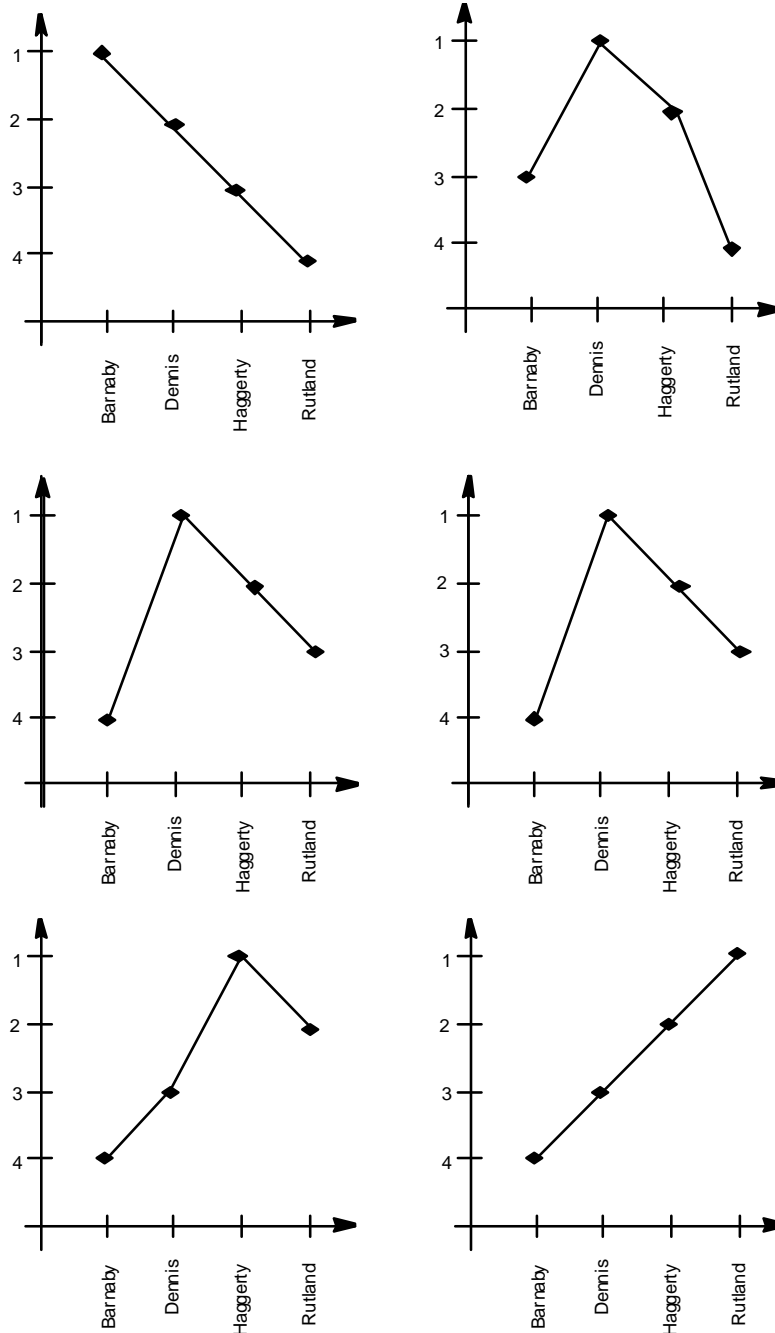
- (e) Could those who ranked Rutland first have achieved a preferable outcome in an election using Borda's method by voting strategically if the others voted as shown in the table? Explain.

Those who voted for Rutland first prefer Haggerty to Dennis. If these five voters rank Haggerty first and Dennis fourth, the Borda counts for Haggerty and Dennis become

$$\begin{aligned} \text{Dennis} &= 9 \times 4 + 4 \times 3 + 3 \times 2 + 5 \times 1 = 59 \\ \text{Haggerty} &= 8 \times 4 + 9 \times 3 + 4 \times 2 + 0 \times 1 = 67 \end{aligned}$$

So, yes, those who ranked Rutland first can achieve a preferable outcome in an election using Borda's method by voting strategically if the others voted as shown in the table.

- (f) Are the preference rankings single-peaked with respect to the order Barnaby, Dennis, Haggerty, Rutland?



From the distributions, we see that the preference rankings are single-peaked.

	Number of Voters					
	4	2	1	6	3	5
Barnaby	1√	3	4	4	4	4
Dennis	2√	1√	1√	1√	3	3√
Haggerty	3	2	2√	2√	1√	2√
Rutland	4	4	3√	3	2√	1√

(g) Which candidate, if any, is the Condorcet winner?

Since the preference rankings are single-peaked, there is a Condorcet winner. We can find the winner by looking at who the middle voter, or ninth voter, voted for. That would be Dennis. So, Dennis is the Condorcet winner.

(h) Which candidate would win an approval vote?

$$\begin{aligned}
 \text{Barnaby} &= 4 \\
 \text{Dennis} &= 18 \\
 \text{Haggerty} &= 15 \\
 \text{Rutland} &= 9
 \end{aligned}$$

So, Dennis is the winner.

(i) Could those who ranked Rutland first have achieved a preferable outcome in an election using the approval method by voting strategically if the others voted as shown in the table? Explain.

Again, those who ranked Rutland first prefer Haggerty to Dennis. If they choose not to approve of Dennis, the tallies would be

$$\begin{aligned}
 \text{Barnaby} &= 4 \\
 \text{Dennis} &= 13 \\
 \text{Haggerty} &= 15 \\
 \text{Rutland} &= 9
 \end{aligned}$$

and Haggerty would be the winner. So, yes, those who ranked Rutland first can achieve a preferable outcome in an election using the approval method by voting strategically if the others voted as shown in the table