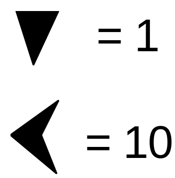
1 Ancient "Positional" Systems

We have already discussed positional systems. We will now look at two examples of ancient positional systems, the Babylonian system and the Mayan system.

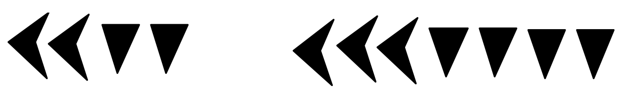
1.1 The Babylonian System

The Babylonian system is a base 60 system, meaning each position represents a power of 60. From what we learned before, this means we need to have symbols that represent digits from 0 to 59. However, the Babylonians did not have a symbol for zero. Therefore, the Babylonians did not have a true positional system. Also, instead of using 59 different symbols to represent the digits 1 to 59, they used combinations of two different symbols.

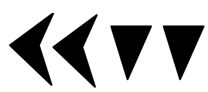


1.2 Converting from Babylonian to Hindu-Arabic

Since the Babylonians used multiple symbols to represent digits in their system, and since they used no zero symbol, the Babylonian representation of numbers can get quite confusing. To cut down on the confusion, we will put in space between each position. For example, in the numeral below



the space between divides the 60's position from the 1's position. In other words, the symbols

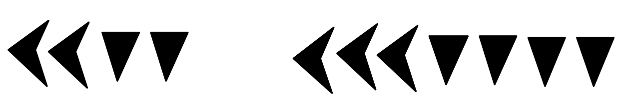


are in the 60's position, and the symbols

are in the 1's position. Since

represents 10 and

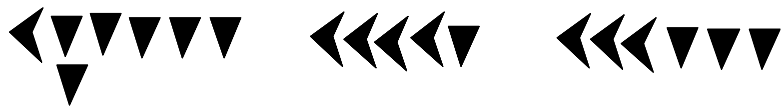
represents 1, the number represented by



is

$$22 \times 60 + 34 \times 1 = 1354$$

Example 1 *Convert*



to Hindu-Arabic.

We see 16 in the 60^2's position, 41 in the 60's position, and 33 in the 1's position. So, the value represented is

$$16 \times 60^2 + 41 \times 60 + 33 = 60,093$$

1.3 Converting from Hindu-Arabic to Babylonian numerals

We convert from base 10 to base 60 in exactly the same way we convert to any other base that is different from 10. We divide the given number by 60 until we cannot divide evenly any more.

Example 2 *Convert 1352 to Babylonian numerals.*

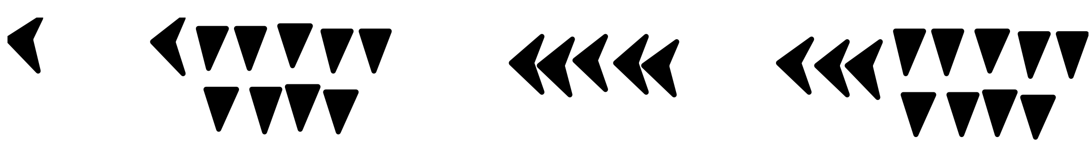
Dividing successively by 60, we obtain

```
      0 R 22
  60)22 R 32
60)1352
```

So, we put 22 in the 60's position and 32 in the 1's position using Babylonian symbols. So, we get

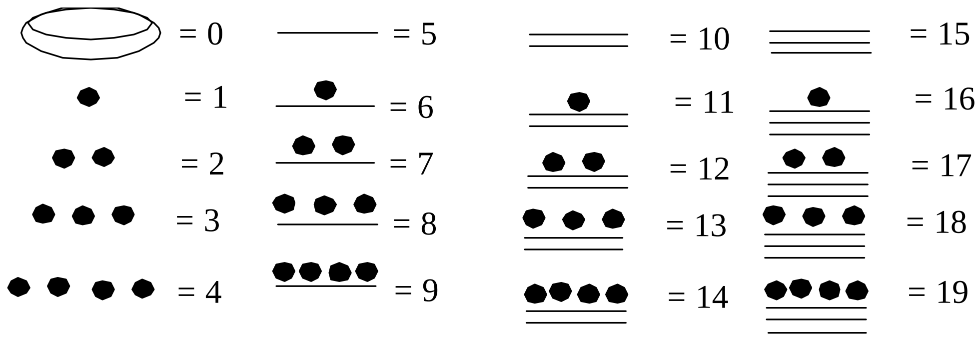
Example 3 *Convert 287,439 to Babylonian numerals.*

```
          0 R 1
       60)1 R 19
     60)79 R 50
   60)4790 R 39
60)287439
```



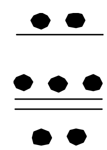
1.4 Converting from Mayan numerals to Hindu-Arabic

The Mayans worked with a base 20 system...almost. They did have a symbol for zero; however, not every position represents a power of 20. One of the positions is the 360's position, or the 18×20's position. Since not every position represents a power of the same number, the Mayan system is also not a true positional system.
Here are the symbols the Mayans used.



The Mayans wrote their numerals vertically, with the bottom position representing 1's and the next position up representing 20's. The third position (counting upward) is the strange position. It is the 18×20's position.. After that, we have the 18×20^2's position, the 18×20^3's position, and so on.

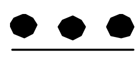
Example 4 *Convert*



to Hindu-Arabic.

The symbol

represents 7, the symbol

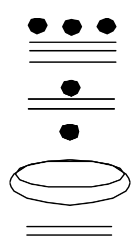


represents 13, and the symbol

represents 2. The 7 is in the 18×20's position, the 13 is in the 20's position, and the 2 is in the 1's position. So, the value of the number represented is

$$7 \times 18 \times 20 + 13 \times 20 + 2 = 2782$$

Example 5 *Convert*



to Hindu-Arabic.

$$18 \times 18 \times 20^3 + 11 \times 18 \times 20^2 + 1 \times 18 \times 20 + 0 \times 20 + 15 = 2,671,575$$

1.5 Converting from Hindu-Arabic to Mayan numerals

In the Mayan numeration system, each position represents a power of 20 (or 18×20 for the special case). So, each time we are going to divide by 20 (with the lone exception of dividing by 18). Let's illustrate this by example.

Example 6 *Convert 1327 to Mayan numerals.*

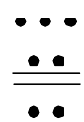
We need to find out the part of 1327 that 20 does not go into even once. Dividing 1327 by 20 gives

$$\begin{array}{r} 66 \text{ R } 7 \\ 20\overline{)1327} \\ \underline{120} \\ 127 \\ \underline{120} \\ 7 \end{array}$$

So, the remainder of 7 represents the part of 1327 that 20 did not go into even once. This is the number that goes in the ones position.
Now, we need to know what goes into the 20's position. The next position above the 20's position is the (18×20)'s position. So, we need to know how many times 18 goes into 66. Remember that 66 represents how many 20's are in 1327. That is why we are dividing the 66 rather that the 1327 at this stage - we are interested in what goes in the 20's position.

$$\begin{array}{r} 3 \text{ R } 12 \\ 18\overline{)66} \\ \underline{54} \\ 12 \end{array}$$

So, the remainder of 12 represents the part of 1327 that 20 went into once, but 18 did not go into. This is the number that goes in the 20's position.
Finally, we need to know what goes into the (18×20)'s position. The next position above the (18×20)'s position is the $\left(18 \times 20^2\right)$'s position. So, we need to know how many times 20 goes into 3. Well, this is easy, 20 goes into 3 zero times with a remainder of 3. This remainder of 3 represents the part of 1327 that 18 went into and 20 went into once, but that 20 did not go into twice. This is what goes in the (18×20)'s position.
Here is the Mayan numeral representation for 1327.



Example 7 *Convert 1,650,615 to Mayan numerals.*

First, we divide 1,650,615 by 20.

$$\begin{array}{r} 82530 \text{ R } 15 \\ 20\overline{)1650615} \end{array}$$

We could perform the long division, but there is an easier way to get at the remainder by using your calculator. By your calculator, you should see $1650615 \div 20 = 82530.75$. The 82530 is the quotient, and the 0.75 part represents the remainder. If we take 0.75×20, we get 15, which is the remainder. So, the keystrokes on your calculator you would use to find the remainder are

1650615/20 ENTER 82530.75

−82530 ENTER 0.75

∗20 ENTER 15

So, 15 goes in the 1's position.
Next, we divide 82,530 by 18. We get 4585 with a remainder of 0. So, 0 goes in the 20's position.
Next, we divide 4585 by 20. On your calculator, you should get 229.25. Going through the keystrokes described above, you find that the remainder is 5. This goes in the (18×20)'s position.
Next, we divide 229 by 20. On your calculator, you should get 11.45. This gives a remainder of 9. This goes in the $\left(18 \times 20^2\right)$'s position.
Finally, we divide 11 by 20 to get 0 with a remainder of 9. So, 11 goes in the $\left(18 \times 20^3\right)$'s position.
Here is the Mayan numeral representation for 1,650,615.