MA 140 - Quiz 1 - Spring 2008

Calculators Allowed

____, come on down!

1. Let $f(x) = x^2 + \ln x$.

   (a) Compute the slope of the secant line through the point $(1, f(1))$ and $(0.9, f(0.9))$.

   $$\begin{aligned} m_{\text{sec}} &= \frac{f(1) - f(0.9)}{1 - 0.9} \\ &= \frac{(1 + \ln 1) - \left(0.9^2 + \ln 0.9\right)}{0.1} \\ &\approx 2.9536 \end{aligned}$$

   (b) Compute the slope of the secant line through the point $(1, f(1))$ and $(0.99, f(0.99))$.

   $$\begin{aligned} m_{\text{sec}} &= \frac{f(1) - f(0.99)}{1 - 0.99} \\ &= \frac{(1 + \ln 1) - \left(0.99^2 + \ln 0.99\right)}{0.01} \\ &\approx 2.995 \end{aligned}$$

   (c) Compute the slope of the secant line through the point $(1, f(1))$ and $(1.01, f(1.01))$.

   $$\begin{aligned} m_{\text{sec}} &= \frac{f(1) - f(1.01)}{1 - 1.01} \\ &= \frac{(1 + \ln 1) - \left(1.01^2 + \ln 1.01\right)}{-0.01} \\ &\approx 3.005 \end{aligned}$$

   (d) Compute the slope of the secant line through the point $(1, f(1))$ and $(1.1, f(1.1))$.

   $$\begin{aligned} m_{\text{sec}} &= \frac{f(1) - f(1.1)}{1 - 1.1} \\ &= \frac{(1 + \ln 1) - \left(1.1^2 + \ln 1.1\right)}{-0.1} \\ &\approx 3.053 \end{aligned}$$

   (e) Use your results from parts (a)-(d) to estimate the slope of $y = f(x)$ at $x = 1$.
   From the above values, it appears that the slope of $f(x) = x^2 + \ln x$ at $x = 1$ is 3.

2. Let $f(x) = x^3 + 3x - 2$.

(a) Show that $f(x)$ has at least one real root.

Since $f(x)$ is a polynomial, it is continuous everywhere. Now, $f(0) = -2 < 0$ and $f(1) = 2 > 0$. So, the Intermediate Value Theorem guarantees that there is a c between 0 and 1 such that $f(c) = 0$. Thus, f has at least one real root.

(b) Using the Method of Bisections, find an interval of length $\frac{1}{32}$ that contains this root.

a	b	$f(a)$	$f(b)$	midpoint	$f(\text{midpoint})$
0	1	-2	2	0.5	$f(0.5) = -0.375$
0.5	1	-0.375	2	0.75	$f(0.75) = 0.671875$
0.5	0.75	-0.375	0.671875	0.625	$f(0.625) \approx 0.11915$
0.5	0.625	-0.375	0.11915	0.5625	$f(0.5625) \approx -0.1345$
0.5625	0.625	-0.1345	0.11915	0.59375	$f(0.59375) = -0.00943$

So, the root of $f(x) = x^3 + 3x - 2$ lies in the interval $(0.59375, 0.625)$.