I expect that you can complete the following problems without the help of a book, notes, friend, or classmate. If there are any problems that cause you difficulty, I strongly encourage you to see me immediately.

1.) Write the equation of the line passing through the point \((2, -3)\) and perpendicular to the line \(4x - 5y = 10\).

\[
-5y = -4x + 10 \\
y = \frac{4}{5}x - 2 \\
\text{perpendicular slope: } -\frac{5}{4} \\
y + 3 = -\frac{5}{4}(x - 2)
\]

2.) Find the exact value of \(\sin \theta\), \(\cos \theta\), and \(\tan \theta\) of an angle \(\theta\) in standard position that has the point \((2, -3)\) on its terminal side.

\[
2^2 + (-3)^2 = r^2 \\
4 + 9 = r^2 \\
13 = r^2 \\
\sqrt{13} = r
\]

\[
\sin \theta = -\frac{3}{\sqrt{13}} \\
\cos \theta = \frac{2}{\sqrt{13}} \\
\tan \theta = -\frac{3}{2}
\]

3.) Let \(\theta\) be an angle in standard position. In what quadrant is \(\tan \theta\) positive and \(\csc \theta\) negative?

The tangent function is positive in quadrants I and III.
The cosecant function is negative in quadrants III and IV.

III.

4.) Write the following expression in terms of a single logarithm with a coefficient of 1.

\[
6 \log_b x + 7 \log_b y - 5 \log_b z \\
\log_b x^6 + \log_b y^7 - \log_b z^5 \\
\log_b \left( \frac{x^6 y^7}{z^5} \right)
\]
5.) Use the information given about the angle $\theta$, $0 \leq \theta < 2\pi$, to find the exact value of $\sin 2\theta$ given that $\tan \theta = \frac{-5}{6}$, $\frac{\pi}{2} < \theta < \pi$.

\begin{align*}
6^2 + (-5)^2 &= r^2 \\
36 + 25 &= r^2 \\
61 &= r^2 \\
\sqrt{61} &= r \\
\sin \theta &= \frac{5}{\sqrt{61}} \\
\cos \theta &= -\frac{6}{\sqrt{61}} \\
\sin \theta &= \frac{5}{\sqrt{61}} \\
\cos \theta &= -\frac{6}{\sqrt{61}} \\
\sin 2\theta &= 2 \sin \theta \cos \theta \\
\sin 2\theta &= 2 \left( \frac{5}{\sqrt{61}} \right) \left( -\frac{6}{\sqrt{61}} \right) \\
\sin 2\theta &= \frac{-60}{61} \\
\end{align*}

6.) Give the exact value of $\tan^{-1}\left( -\frac{1}{\sqrt{3}} \right)$.

The inverse tangent function is restricted to quadrants I and IV. Tangent values are negative in the fourth quadrant. Based on the ratio of the sides of the reference triangle, it must be a 30-60-90 triangle. Thus, $\tan^{-1}\left( -\frac{1}{\sqrt{3}} \right) = -\frac{\pi}{6}$.

\begin{align*}
\left( \sqrt{3} \right)^2 + (-1)^2 &= r^2 \\
3 + 1 &= r^2 \\
4 &= r^2 \\
2 &= r \\
\end{align*}

7.) Factor the following polynomial completely.

\begin{align*}
2x^3 + 54 \\
2(x^3 + 27) \\
2(x+3)(x^2 - 3x + 9) \\
\end{align*}
8.) Simplify the following complex fraction. 

\[
\frac{2\ \frac{3}{x}}{\frac{4\ +\ \frac{7}{y} - \frac{2\ \frac{3}{x}}{\frac{4\ +\ \frac{7}{y} - \frac{2\ \frac{3}{x}}{\frac{4\ +\ \frac{7}{y}}{xy}}}}}}}
\]

9.) What is the domain of \( f(x) = \frac{2}{x + 5} \)? Explain your answer, and express it in interval notation.

Any number may be plugged in for \( x \) except for -5. If -5 is plugged in, \( f(x) \) is undefined. So, the domain is all real numbers except -5. In interval notation, that is \((-\infty, -5) \cup (-5, \infty)\).

10.) Find \( f \circ g \) using \( f(x) = 3x + 4 \) and \( g(x) = 5x^2 - 7 \).

\[
f \circ g = f(g(x)) \\
= f(5x^2 - 7) \\
= 3(5x^2 - 7) + 4 \\
= 15x^2 - 21 + 4 \\
f \circ g = 15x^2 - 17
\]

11.) Find functions \( f \) and \( g \) so that \( f \circ g = H \) where \( H(x) = \sqrt[3]{x^2 - 4} \).

Answers may vary.
Let \( f(x) = \sqrt[3]{x} \) and \( g(x) = x^2 - 4 \).

Then \( f \circ g = f(g(x)) = f(x^2 - 4) = \sqrt[3]{x^2 - 4} \).

12.) Factor the following polynomial completely.

\[
3x^4 - 30x^2 + 27 \\
3(x^4 - 10x^2 + 9) \\
3(x^2 - 1)(x^2 - 9) \\
3(x + 1)(x - 1)(x + 3)(x - 3)
\]
13.) Use \( \cos x = -\frac{15}{17} \) and \( x \) is a second quadrant angle to find \( \sin x \).

\[
(-15)^2 + y^2 = 17^2
\]

\[
225 + y^2 = 289
\]

\[
y^2 = 64
\]

\[
y = 8
\]

\[
\sin x = \frac{8}{17}
\]

14.) Solve the following equation.

\[
\log_2 (x - 3) = 4
\]

Change to exponential form.

\[
2^4 = x - 3
\]

\[
16 = x - 3
\]

\[
x = 19
\]

\[
\log_2 (19 - 3) = 4
\]

\[
\log_2 (16) = 4
\]

\[
4 = 4
\]